

Math 1010: Solutions to Practice Questions for Unit #4

1. A neighbourhood pub proposal outlines the following information:

- The average customer spends \$15 on food and drinks.
- Fixed costs are \$10,000 per month.
- Food and drinks are priced at 3 times their cost.
- Capacity is estimated at 50 customers per day. Assume 24 business days per month.

Information given:

- $S=15$
- $FC=10,000$
- Price of food and drinks (S)= $3C$. Since $S=15$, $3C=15$, so $C=5$.
- Capacity is 1,200 people per month (50 customers * 24 days = 1,200)

Set up our Revenue and Cost functions:

$$TR = 15x$$

$$TC = 5x + 10,000$$

$$NI = TR - TC$$

$$= 15x - (5x + 10,000)$$

$$= 15x - 5x - 10,000$$

$$= 10x - 10,000$$

a. Calculate the break-even point i) in units ii) in dollars and iii) as a percent of capacity.

Break-even point in units when $NI=0$. Solve for x .

$$\begin{aligned} 0 &= 10x - 10,000 \\ 10,000 &= 10x - 10,000 + 10,000 \\ \frac{10,000}{10} &= \frac{10x}{10} \\ x &= 1,000 \end{aligned}$$

Break-even volume = 1,000 customers

Break-even point in dollars. X is 1,000 customers

$$\begin{aligned} TR &= 15x \\ &= 15(1,000) \\ &= \$15,000 \end{aligned}$$

Break even in dollars = \$15,000

Break-even as a percent of capacity

$$\begin{aligned} &\frac{BE \text{ volume}}{Capacity} \times 100 \\ &= \frac{1,000}{1,200} \times 100 \end{aligned}$$

$$= 83.33\%$$

b. Determine the net income at a sales volume of i) 1,200 customers, ii) \$9,750

To calculate the net income for 1,200 customers, $x=1,200$

$$\begin{aligned} NI &= 10x - 10,000 \\ &= 10(1,200) - 10,000 \\ &= 12,000 - 10,000 \\ &= \$2,000 \end{aligned}$$

The net income is \$2,000

To calculate the net income for \$9,750:

1. Calculate how many customers generate that total revenue
2. Calculate the net income for that many customers

$$\begin{aligned} TR &= 15x \\ 9,750 &= 15x \\ x &= 650 \end{aligned}$$

650 customers generate a total revenue of \$9,750.

$$\begin{aligned} NI &= 10x - 10,000 \\ &= 10(650) - 10,000 \\ &= 6,500 - 10,000 \\ &= -\$3,500 \end{aligned}$$

650 customers leads to a net loss of \$3,500.

c. Determine the level of operations as a percent of capacity to generate a net income of \$1,250.

Determine how many customers provide a net income of \$1,250.

$$\begin{aligned} NI &= 10x - 10,000 \\ 1,250 &= 10x - 10,000 \\ 1,250 + 10,000 & \\ &= 10x - 10,000 + 10,000 \\ \frac{11,250}{10} &= \frac{10x}{10} \\ x &= 1,125 \end{aligned}$$

1,125 customers will generate a net income of \$1,250

Calculate as a percentage of capacity

$$\frac{1,125}{1,200} \times 100 = 93.75\%$$

d. Determine the level of operations in dollars to sustain a loss of no more than \$1,500.

Determine how many customers provide a net income of -\$1,500.

$$\begin{aligned} NI &= 10x - 10000 \\ -1500 &= 10x - 10000 \\ -1500 + 10000 &= 10x - 10000 + 10000 \\ 8500 &= 10x \\ x &= 850 \end{aligned}$$

Determine the total revenue that leads to a loss of \$1,500.

$$\begin{aligned} &850 \text{ customers will generate a loss of } \$1,500 \\ TR &= 15x \\ &= 15(850) \\ &= \$12,750 \end{aligned}$$

At \$12,750 in sales, there would be a \$1,500 loss.

e. Determine the break-even point in dollars if fixed costs are reduced by 20%.

Reduce the fixed costs by 20%

$$\begin{aligned} FC &= 10,000(1 - 0.20) \\ &= 10,000(0.80) \\ &= 8,000 \end{aligned}$$

Create a new NI formula. For break-even, NI=0 and solve for x.

$$\begin{aligned} NI &= 10x - 8,000 \\ 0 &= 10x - 8,000 \\ 8,000 &= 10x \\ x &= 800 \end{aligned}$$

800 customers would provide the break-even volume when fixed costs are reduced by 20%

Calculate the revenue for the break-even point in customers

$$\begin{aligned} TR &= 15x \\ &= 15(800) \\ &= \$12,000 \end{aligned}$$

At \$12,000, the pub will break even if the fixed costs are reduced by 20%

f. Determine the break-even point in dollars if fixed costs are reduced to \$7,500 and the food and drinks are priced at 5 times their cost.

Find new values for fixed costs (FC), selling price (S), total cost, and total revenue

$$\begin{aligned} FC &= \$7,500 \\ S &= 5 \cdot C = 5 \cdot 5 = \$25 \\ TR &= 25x \\ TC &= 5x + 7,500 \end{aligned}$$

Create a new formula for the net income

$$\begin{aligned} NI &= TR - TC \\ &= 25x - (5x + 7,500) \\ &= 20x - 7,500 \end{aligned}$$

Calculate how many customer are required to break even in sales. $NI=0$

$$\begin{aligned}0 &= 20x - 7,500 \\20x &= 7,500 \\x &= 375\end{aligned}$$

375 customers will be required for the pub to break even at the new costs

Calculate the total revenue at break-even

$$\begin{aligned}TR &= 25x \\&= 25(375) \\&= \$9,375\end{aligned}$$

At \$9,375 in sales, the pub would break even.

2. Sunshine Tours offers tours for \$1,000 each. It can provide a maximum of 1,000 tours per year. The variable costs for each tour are \$300 for meals and accommodation, \$150 for taxi transfers and \$200 in other miscellaneous activities. The fixed overhead costs for the company are \$210,000 per year.

Information given:

- $S = 1000$
- $VC=300+150+200=650$
- $FC=210,000$
- Maximum = 1,000 tours

Set up our Revenue and Cost functions:

$$TR = 1,000x$$

$$TC = 650x + 210,000$$

$$NI = 1,000x - (650x + 210,000)$$

$$NI = 350x - 210,000$$

a. Use algebra to calculate the break-even point in volume and in sales.

$$NI = 350x - 210,000$$

$$0 = 350x - 210,000$$

$$210,000 = 350x$$

$$x = \mathbf{600 \text{ units (volume)}}$$

$$TR = 1,000x$$

$$= 1,000(600)$$

$$\mathbf{TR = \$600,000 \text{ sales}}$$

b. What is the break-even as a percent of the capacity?

Capacity is 1,000 units, break-even is 600 units.

$$\% \text{ of Capacity} = \frac{600}{1000} \times 100\% = 60\%$$

3. Choco Corp. manufactures boxes of chocolate candies. Its basic cost and profit information is summarized below:

- Selling price (per box): \$8.00 [This is S]
- Variable cost (per box): \$2.00 [This is VC]
- Fixed cost (total per month): \$24,000 [This is FC]

The company has the capacity to manufacture up to 10,000 boxes of its candy per month.

Set up our Revenue and Cost functions:

$$TR = 8x$$

$$TC = 2x + 24,000$$

$$NI = 8x - (2x + 24,000)$$

$$NI = 6x - 24,000$$

a. What is the break-even point in terms of number of units and in dollars of sales?

$$NI = 6x - 24,000$$

$$0 = 6x - 24,000$$

$$24,000 = 6x$$

$$x = 4,000 \text{ boxes (volume)}$$

$$TR = 8x$$

$$= 8(4,000)$$

$$= \$32,000 \text{ sales}$$

b. What is the necessary sales volume (in number of units and in sales dollars) for Choco Corp. to earn \$12,000 in net income?

$$NI = \$12,000$$

$$12,000 = 6x - 24,000$$

$$36,000 = 6x$$

$$x = 6,000 \text{ boxes (volume)}$$

$$TR = 8x$$

$$= 8(6,000)$$

$$= \$48,000 \text{ sales}$$

c. What is the necessary sales volume (in number of units and in sales dollars) for the company to earn a profit equal to 20 percent of its revenues?

20% of revenue
 $= 0.20(TR)$
 $= 0.20 \times 8x$
 $= 1.6x$ is the new net income

$1.6x = 6x - 24,000$
 $24,000 = 6x - 1.6x$
 $24,000 = 4.4x$
 $x = 5,455$ (rounded up)

$TR = 8x$
 $= 8(5,455)$
 $= \$43,640$ (sales volume)

d. The primary material used to produce the candy is obtained from a foreign supplier. War in the region has created problems with shipping the product to Choco, and as a result, materials cost is expected to increase. The impact is that the materials cost will increase by \$0.80 per box of candy produced. How many additional units must the company sell in order to break even?

VC increase by \$0.80, therefore, VC=\$2.80

New net income:
 $NI = 8x - 2.8x - 24,000$
 $NI = 5.2x - 24,000$

New break-even:
 $NI = 5.2x - 24,000$
 $0 = 5.2x - 24,000$
 $24,000 = 5.2x$
 $x = 4615.38$ (round up to 4,616 units)

New break-even volume less original break-even:
4,616-4,000=616 more units to break even

4. A firm manufactures a product that sells for \$12 per unit. Variable cost per unit is \$8 and fixed cost per period is \$1,200. Capacity per period is 1,000 units.

Information given:

- $S = \$12$
- $VC = \$8$
- $FC = \$1,200$

- Capacity is 1,000 units

a. Write algebraic statements of the revenue function and the cost function.

$$TR = 12x$$

$$TC = 8x + 1,200$$

b. Compute the break-even point in units, as a percent of capacity and in sales dollars.

$$NI = 12x - (8x + 1,200) = 4x - 1,200$$

At break-even, $NI=0$

$$0 = 4x - 1,200$$

$$1,200 = 4x$$

$$x = 300 \text{ units}$$

As a percentage of capacity:

$$\frac{300}{1,000} \times 100 = 30\%$$

$$TR = 12x$$

$$= 12(300)$$

$$= \$3,600 \text{ in sales}$$

5. Engineering estimates indicate the variable cost of manufacturing a new product will be \$35 per unit. Based on market research, the selling price of the product is to be \$120 per unit and variable selling expense is expected to be \$15 per unit. The fixed costs applicable to the new product are estimated to be \$2,800 per period and capacity per period is 100 units.

Information given:

- $S = \$120$
- $VC = \$35 + 15 = 50$
- $FC = \$2,800$
- max 100 units

a. Write algebraic statements of the revenue function and the cost function.

$$TR = 120x$$

$$TC = 50x + 2,800$$

b. Compute the break-even point in units, as a percent of capacity and in sales dollars.

$$NI = 120x - (50x + 2,800) = 70x - 2,800$$

At break-even, $NI=0$

$$0 = 70x - 2,800$$

$$2,800 = 70x$$

$$x = 40 \text{ units}$$

As a percentage of capacity:

$$\frac{40}{100} \times 100 = \mathbf{40\%}$$

$$TR = 120x$$

$$= 120(40)$$

$$= \$4,800 \text{ in sales}$$

c. Determine the net income at a volume of 70 units.

$$NI = 70x - 2,800$$

$$= 70(70) - 2,800$$

$$= 4,900 - 2,800$$

$$= \$2,100$$

d. Determine the net income at a volume of 30% of capacity.

$$30\% \text{ of capacity} = 0.30(100) = 30 \text{ units}$$

$$NI = 70x - 2,800$$

$$= 70(30) - 2,800$$

$$= 2,100 - 2,800$$

$$= -\$700 \text{ (a loss of \$700)}$$

e. Determine the net income at a sales volume of \$10,200.

Need to calculate how many units give a total revenue of \$10,200

$$TR = 120x$$

$$10,200 = 120x$$

$$x = 85 \text{ units}$$

$$NI = 70x - 2,800$$

$$= 70(85) - 2,800$$

$$= 5,950 - 2,800$$

$$= \$3,150$$

f. Determine the level of operations as a percentage of capacity to generate a net income of \$2,240.

Find the number of units that generate $NI = 2,240$

$$\begin{aligned}
2,240 &= 70x - 2,800 \\
2,240 + 2,800 &= 70x - 2,800 + 2,800 \\
5,040 &= 70x \\
x &= 72 \text{ units}
\end{aligned}$$

As a percentage of capacity:

$$\frac{72}{100} \times 100 = \mathbf{72\%}$$

g. Determine the number of units that must be sold to make a net income of \$1,050.

$$\begin{aligned}
NI &= 70x - 2,800 \\
1,050 &= 70x - 2,800 \\
1,050 + 2,800 &= 70x - 2,800 + 2,800 \\
3,850 &= 70x \\
x &= 55 \text{ units}
\end{aligned}$$

h. Determine the sales volume in dollars to suffer a loss of no more than \$350.

$$\begin{aligned}
NI &= 70x - 2,800 \\
-350 &= 70x - 2,800 \\
-350 + 2,800 &= 70x - 2,800 + 2,800 \\
2,450 &= 70x \\
x &= 35 \text{ units}
\end{aligned}$$

$$\begin{aligned}
TR &= 120x \\
&= 120(35) \\
&= \$4,200 \text{ in sales}
\end{aligned}$$

i. Determine the break-even point in units if fixed costs are increased to \$3,150.

$$\text{New NI formula: } NI = 70x - 3,150$$

$$\begin{aligned}
0 &= 70x - 3,150 \\
3,150 &= 70x \\
x &= 45 \text{ units}
\end{aligned}$$

j. Determine the break-even point as a percentage of capacity if fixed costs are reduced by \$160 and the variable cost of manufacturing is increased to \$39 per unit.

$$\begin{aligned}
\text{New FC} &= 2,800 - 160 = \$2,640 \\
\text{New VC} &= 39 + 15 = \$54 \\
\text{New NI} &= 120x - (54x + 2,640) = 66x - 2,640
\end{aligned}$$

$$NI = 66x - 2,640$$

$$0 = 66x - 2,640$$

$$2,640 = 66x$$

$$x = 40$$

As a percentage of capacity:

$$\frac{40}{100} \times 100 = \mathbf{40\%}$$

k. Determine the break-even point in dollars if the selling price is reduced to \$100 per unit.

$$S=100, \text{ so } TR=100x$$

$$\text{New NI} = 100x - (50x + 2,800) = 50x - 2,800$$

$$0 = 50x - 2,800$$

$$2,800 = 50x$$

$$x = 56 \text{ units}$$

$$TR = 100(56)$$

$$= \$5,600$$

6. The lighting division of Universal Electric Company plans to introduce a new streetlight based on the following accounting information.

- Fixed costs per period are \$3,136
- Variable cost per unit is \$157
- Selling price per unit is \$185
- Capacity per period is 320 units

a. Write algebraic statements of the revenue function and the cost function.

$$TR=185x$$

$$TC=157x+3,136$$

b. Compute the break-even point in units, as a percentage of capacity and in dollars.

$$NI = 185x - (157x + 3,136) = 28x - 3,136$$

At break-even, NI=0

$$0 = 28x - 3,136$$

$$3,136 = 28x$$

$$x = 112 \text{ units}$$

As a percentage of capacity:

$$\frac{112}{320} \times 100 = \mathbf{35\%}$$

$$\begin{aligned} TR &= 185x \\ &= 185(112) \\ &= \$20,720 \text{ in sales} \end{aligned}$$

c. Determine the net income at a sales volume of

i) 25% of capacity

Need to find the number of units: 25% of 320=80 units

$$\begin{aligned} NI &= 28x - 3,136 \\ &= 28(80) - 3,136 \\ &= 2,240 - 3,136 \\ &= -\$896 \text{ (loss)} \end{aligned}$$

ii) \$24,975

$$\begin{aligned} TR &= 24,975 \\ 24,975 &= 185x \\ x &= 135 \text{ units} \end{aligned}$$

$$\begin{aligned} NI &= 28x - 3,136 \\ &= 28(135) - 3,136 \\ &= 3,780 - 3,136 \\ &= \$644 \end{aligned}$$

d. Determine the level of operations as a percentage of capacity to generate a net income of:

i) \$2,240

$$\begin{aligned} NI &= 28x - 3,136 \\ 2,240 &= 28x - 3,136 \\ 2,240 + 3,136 &= 28x - 3,136 + 3,136 \\ 5,376 &= 28x \\ x &= 192 \text{ units} \end{aligned}$$

As a percentage of capacity:

$$\frac{192}{320} \times 100 = \mathbf{60\%}$$

ii) \$6,720

$$\begin{aligned} NI &= 28x - 3,136 \\ 6,720 &= 28x - 3,135 \end{aligned}$$

$$\begin{aligned}
 6,720 + 3,136 &= 28x - 3,136 + 3,136 \\
 9,856 &= 28x \\
 x &= 352 \text{ units}
 \end{aligned}$$

As a percentage of capacity:

$$\frac{352}{320} \times 100 = \mathbf{110\%}$$

This is impossible, since the maximum capacity is 320 units.

e. Determine the break-even point as a percentage of capacity

i) if fixed costs are reduced to \$2,688

$$\begin{aligned}
 \text{New NI} &= 28x - 2,688 \\
 0 &= 28x - 2,688 \\
 2,688 &= 28x \\
 x &= 96 \text{ units}
 \end{aligned}$$

As a percentage of capacity:

$$\frac{96}{320} \times 100 = \mathbf{30\%}$$

ii) if fixed costs increase to \$4,588 and variable costs are reduced to 80% of the selling price

$$\begin{aligned}
 \text{New FC} &= 4,588 \\
 \text{New VC} &= 0.80(185) = 148 \\
 \text{New NI} &= 185x - (148x + 4,588) = 37x - 4,588
 \end{aligned}$$

$$\begin{aligned}
 0 &= 37x - 4,588 \\
 4,588 &= 37x \\
 x &= 124 \text{ units}
 \end{aligned}$$

As a percentage of capacity:

$$\frac{124}{320} \times 100 = \mathbf{38.75\%}$$

iii) if the selling price is reduced to \$171

$$\text{New NI} = 171x - (157x + 3,136) = 14x - 3,136$$

$$\begin{aligned}
 0 &= 14x - 3,136 \\
 3,136 &= 14x \\
 x &= 224 \text{ units}
 \end{aligned}$$

As a percentage of capacity:

$$\frac{224}{320} \times 100 = 70\%$$

7. A manufacturer of major appliances provides the following information about the operations of the refrigeration division.

- Fixed costs per period are \$26,880
- Variable costs per unit are \$360
- Selling price per unit is \$640
- Capacity is 150 units.

a. Write algebraic statements of the revenue function and the cost function.

$$TR = 640x$$

$$TC = 360x + 26,880$$

b. Compute the break-even point in units, as a percentage of capacity and in dollars.

$$NI = 640x - (360x + 26,880) = 280x - 26,880$$

At break-even, $NI = 0$

$$0 = 280x - 26,880$$

$$26,880 = 280x$$

$$x = 96 \text{ units}$$

As a percentage of capacity:

$$\frac{96}{150} \times 100 = \mathbf{64\%}$$

$$TR = 640x$$

$$= 640(96)$$

$$= \$61,440 \text{ in sales}$$

c. Determine the net income at a sales volume of

i) \$78,080

$$TR = 78,080$$

$$78,080 = 640x$$

$$x = 122 \text{ units}$$

$$NI = 280x - 26,880$$

$$= 280(122) - 26,880$$

$$= 34,160 - 26,880$$

$$= \$7,280$$

ii) 48% of capacity

Need to find the number of units: 48% of 150=72 units

$$\begin{aligned} NI &= 280x - 26,880 \\ &= 280(72) - 26,880 \\ &= 20,160 - 26,880 \\ &= -\$6,720 \text{ (loss)} \end{aligned}$$

d. Determine the volume of output as a percentage of capacity to generate a net income of \$4,200.

$$\begin{aligned} NI &= 280x - 26,880 \\ 4,200 &= 280x - 26,880 \\ 4,200 + 26,880 &= 280x - 26,880 + 26,880 \\ 31,080 &= 280x \\ x &= 111 \end{aligned}$$

As a percentage of capacity:

$$\frac{111}{150} \times 100 = \mathbf{74\%}$$

e. Determine the break-even point in dollars if fixed costs are increased to \$32,200.

$$\text{New NI} = 280x - 32,200$$

$$\begin{aligned} NI &= 280x - 32,200 \\ 0 &= 280x - 32,200 \\ 32,200 &= 280x \\ x &= 115 \text{ units} \end{aligned}$$

$$\begin{aligned} TR &= 640x \\ &= 640(115) \\ &= \$73,600 \end{aligned}$$

f. Determine the break-even point as a percentage of capacity if fixed costs are reduced to \$23,808 while variable costs are increased to 60% of sales.

$$\begin{aligned} \text{New FC} &= 23,808 \\ \text{New VC} &= 0.60(640) = 384 \\ \text{New NI} &= 640x - (384x + 23,808) = 256x - 23,808 \end{aligned}$$

$$\begin{aligned} 0 &= 256x - 23,808 \\ 23,808 &= 256x \\ x &= 93 \text{ units} \end{aligned}$$

As a percentage of capacity:

$$\frac{93}{150} \times 100 = 62\%$$

8. The Superior CD Company sells CDs for \$10 each. Manufacturing cost is \$2.60 per CD; marketing costs are \$2.40 per CD and royalty payments are 20% of the selling price. The fixed cost of preparing the CDs is \$18,000. Capacity is 15,000 CDs.

Information given:

- $S=10$
- $VC=2.60+2.40+0.20(10)=5.00+2.00=\7.00
- $FC=18,000$
- Capacity 15,000 CDs

- a. Write algebraic statements of the revenue function and the cost function.

$$TR=10x$$

$$TC=7x+18,000$$

- b. Compute the break-even point in units, as a percentage of capacity and in dollars.

$$NI = 10x - (7x + 18,000) = 3x - 18,000$$

At break-even, $NI=0$

$$0 = 3x - 18,000$$

$$18,000 = 3x$$

$$x = 6,000 \text{ units}$$

As a percentage of capacity:

$$\frac{6,000}{15,000} \times 100 = \mathbf{40\%}$$

$$TR = 10x$$

$$= 10(6,000)$$

$$= \$60,000 \text{ in sales}$$

- c. Determine the net income at a sales volume of 55% of capacity.

$$55\% \text{ of } 15,000 = 8,250 \text{ units}$$

$$NI = 3x - 18,000$$

$$= 3(8,250) - 18,000$$

$$= \$6,750$$

d. Determine the sales volume in dollars to generate a net income of \$13,800.

$$NI = 13,800$$

$$13,800 = 3x - 18,000$$

$$13,800 + 18,000 = 3x - 18,000 + 18,000$$

$$31,800 = 3x$$

$$x = 10,600$$

$$TR = 10x$$

$$= 10(10,600)$$

$$= \$106,000 \text{ in sales}$$

e. Determine the break-even point in units if fixed costs are increased by \$1,600 while manufacturing cost is reduced by \$0.50 per CD.

$$\text{New FC} = 18,000 + 1,600 = 19,600$$

$$\text{New VC} = 7 - 0.50 = 6.50$$

$$\text{New NI} = 10x - (6.5x + 19,600) = 3.5x - 19,600$$

$$0 = 3.5x - 19,600$$

$$19,600 = 3.5x$$

$$x = 5,600 \text{ units}$$

f. Determine the break-even point in units if the selling price is increased by 10% while fixed costs are increased by \$2,900.

$$\text{New S} = 10 + 0.10(10) = 10 + 1 = 11$$

$$\text{New FC} = 18,000 + 2,900 = 20,900$$

$$\text{New NI} = 11x - (7x + 20,900) = 4x - 20,900$$

$$0 = 4x - 20,900$$

$$20,900 = 4x$$

$$x = 5,225 \text{ units}$$

9. For each of the following, perform a break-even analysis showing:

- algebraic statements of the revenue function and the cost function
- computation of the break-even point in sales dollars and as a percent of capacity

a. The following data pertains to the operating budget of Matt Manufacturing

Sales		\$720,000
Fixed Costs	\$220,000	
Total Variable Costs	324,000	544,000

Net Income

\$176,000

Capacity is a sales volume of \$800,000 per period.

Assume $S=\$1$, therefore sales=720,000 units

$$VC \text{ per unit} = \frac{324,000}{720,000} = 0.45$$

$$TR=x$$

$$TC=0.45x+220,000$$

$$NI = x - (0.45x + 220,000)$$

$$= 0.55x - 220,000$$

$$0 = 0.55x - 220,000$$

$$220,000 = 0.55x$$

$$x = 400,000 \text{ units}$$

$$TR=\$400,000$$

As a percentage of capacity:

$$\frac{400,000}{800,000} \times 100 = \mathbf{50\%}$$

b. A company has compiled the following estimates regarding operations:

Sales		\$120,000
Fixed Costs	\$43,200	
Total Variable Costs	48,000	91,200
Net Income		\$28,800

Capacity is a sales volume of \$150,000.

Assume $S=\$1$, therefore sales=120,000 units

$$VC \text{ per unit} = \frac{48,000}{120,000} = 0.40$$

$$TR=x$$

$$TC=0.40x+43,200$$

$$NI = x - (0.40x + 43,200)$$

$$= 0.60x - 43,200$$

$$0 = 0.60x - 43,200$$

$$43,200 = 0.60x$$

$$x = 72,000 \text{ units}$$

$$TR = \$72,000$$

As a percentage of capacity:

$$\frac{72,000}{150,000} \times 100 = \mathbf{48\%}$$

10. For question 9a) determine:

Information given:

- $TR = x$
- $TC = 0.45x + 220,000$
- $NI = 0.55x - 220,000$

a. the net income at 85% of capacity;

$$85\% \text{ of capacity} = 0.85 \times 800,000 = 680,000 \text{ units}$$

$$\begin{aligned} NI &= 0.55x - 220,000 \\ &= 0.55(680,000) - 220,000 \\ &= 374,000 - 220,000 \\ &= \$154,000 \end{aligned}$$

b. the net income at 30% of capacity;

$$30\% \text{ of capacity} = 0.30 \times 800,000 = 240,000 \text{ units}$$

$$\begin{aligned} NI &= 0.55x - 220,000 \\ &= 0.55(240,000) - 220,000 \\ &= 132,000 - 220,000 \\ &= -\$88,000 \end{aligned}$$

c. the level of operations as a percentage of capacity to realize a net income of \$66,000;

$$\begin{aligned} NI &= 0.55x - 220,000 \\ 66,000 &= 0.55x - 220,000 \\ 66,000 + 220,000 &= 0.55x - 220,000 + 220,000 \\ 286,000 &= 0.55x \\ x &= 520,000 \text{ units} \end{aligned}$$

As a percentage of capacity:

$$\frac{520,000}{800,000} \times 100 = \mathbf{65\%}$$

d. the level of operations in dollars to sustain a loss of no more than \$6,600;

$$\begin{aligned} NI &= 0.55x - 220,000 \\ -6,600 &= 0.55x - 220,000 \\ -6,600 + 220,000 &= 0.55x - 220,000 + 220,000 \\ 213,400 &= 0.55x \\ x &= 388,000 \text{ units} \end{aligned}$$

$$TR = \$388,000$$

e. the break-even point in dollars if fixed costs are increased by 15%; and

$$\begin{aligned} \text{New FC} &= 220,000 + 0.15(220,000) = 220,000 + 33,000 = \$253,000 \\ \text{New NI} &= 0.55x - 253,000 \end{aligned}$$

$$\begin{aligned} 0 &= 0.55x - 253,000 \\ 253,000 &= 0.55x \\ x &= 460,000 \text{ units} \end{aligned}$$

$$TR = \$460,000$$

f. the break-even point as a percentage of capacity if fixed costs are decreased by \$18,400 and variable costs are increased to 52% of sales.

$$\begin{aligned} \text{New FC} &= 220,000 - 18,400 = \$201,600 \\ \text{New VC} &= 0.52 \\ \text{New NI} &= x - (0.52x + 201,600) = 0.48x - 201,600 \end{aligned}$$

$$\begin{aligned} NI &= 0.48x - 201,600 \\ 0 &= 0.48x - 201,600 \\ 201,600 &= 0.48x \\ x &= 420,000 \text{ units} \end{aligned}$$

As a percentage of capacity:

$$\frac{420,000}{800,000} \times 100 = \mathbf{52.5\%}$$

11. The following information is available from the accounting records of Eva Corporation:

- Fixed costs per period are \$4,800
- Sales volume for the last period was \$19,360

- Variable costs were \$13,552
- Capacity per period is a sales volume of \$32,000.

a. Write algebraic statements of the revenue function and the cost function.

Assume $S=\$1$, therefore sales=19,360 units

$$VC \text{ per unit} = \frac{13,552}{19,360} = 0.70$$

$$TR=x$$

$$TC=0.70x+4,800$$

b. Compute the break-even point in dollars and as a percentage of capacity.

$$\begin{aligned} NI &= x - (0.70x + 4,800) \\ &= 0.30x - 4,800 \end{aligned}$$

$$0 = 0.30x - 4,800$$

$$4,800 = 0.30x$$

$$x = 16,000$$

$$TR=\$16,000$$

As a percentage of capacity:

$$\frac{16,000}{32,000} \times 100 = \mathbf{50\%}$$

c. Determine the net income at a sales volume of

i) \$24,000

At $S=\$1$, $x=24,000$ units

$$\begin{aligned} NI &= 0.30x - 4,800 \\ &= 0.30(24,000) - 4,800 \\ &= 7,200 - 4,800 \\ &= \$2,400 \end{aligned}$$

ii) 30% of capacity

30% of 32,000 = 9,600 units

$$\begin{aligned} NI &= 0.30x - 4,800 \\ &= 0.30(9,600) - 4,800 \\ &= 2,880 - 4,800 \end{aligned}$$

$$= -\$1,920$$

d. Determine the level of operations as a percentage of capacity to generate a net income of \$3,264

$$\begin{aligned} NI &= 3,264 \\ 3,264 &= 0.30x - 4,800 \\ 3,264 + 4,800 &= 0.30x - 4,800 + 4,800 \\ 8,064 &= 0.30x \\ x &= 26,880 \end{aligned}$$

As a percentage of capacity:

$$\frac{26,880}{32,000} \times 100 = \mathbf{84\%}$$

e. Determine the break-even point as a percentage of capacity.

i) if fixed costs are decreased by \$600

$$\begin{aligned} \text{New FC} &= 4,800 - 600 = \$4,200 \\ \text{New NI} &= 0.30x - 4,200 \end{aligned}$$

$$\begin{aligned} NI &= 0.30x - 4,200 \\ 0 &= 0.30x - 4,200 \\ 4,200 &= 0.30x \\ x &= 14,000 \text{ units} \end{aligned}$$

As a percentage of capacity:

$$\frac{14,000}{32,000} \times 100 = \mathbf{43.75\%}$$

ii) if fixed costs are increased to \$5,670 and variable costs are changed to 55% of sales

$$\begin{aligned} \text{New FC} &= 5,670 \\ \text{New VC} &= 0.55 \\ \text{New NI} &= x - (0.55x + 5,670) = 0.45x - 5,670 \end{aligned}$$

$$\begin{aligned} NI &= 0.45x - 5,670 \\ 0 &= 0.45x - 5,670 \\ 5,670 &= 0.45x \\ x &= 12,600 \text{ units} \end{aligned}$$

As a percentage of capacity:

$$\frac{12,600}{32,000} \times 100 = \mathbf{39.375\%}$$

12. The operating budget of Bea Company contains the following information:

Sales at 80% capacity		\$400,000
Fixed Costs	\$105,000	
Variable Costs	260,000	365,000
Net Income		\$35,000

Information given:

- Assume $S = \$1$, therefore sales = 400,000 units
- $VC \text{ per unit} = \frac{260,000}{400,000} = 0.65$
- $TR = x$
- $TC = 0.65x + 105,000$
- 80% capacity is 400,000

$$\frac{400,000}{80} = \frac{x}{100}$$

$$80x = 400,000 \times 100$$

$$x = 500,000 \quad (100\% \text{ capacity})$$

a. Compute the break-even point as a percentage of capacity and in dollars.

$$\begin{aligned}
 NI &= x - (0.65x + 105,000) \\
 &= 0.35x - 105,000 \\
 0 &= 0.35x - 105,000 \\
 105,000 &= 0.35x \\
 x &= 300,000
 \end{aligned}$$

As a percentage of capacity:

$$\frac{300,000}{500,000} \times 100 = 60\%$$

$$TR = \$300,000$$

b. Determine the net income at 40% of capacity.

$$40\% \text{ of capacity} = 0.40(500,000) = 200,000 \text{ units}$$

$$\begin{aligned}
 NI &= 0.35x - 105,000 \\
 &= 0.35(200,000) - 105,000 \\
 &= 70,000 - 105,000 \\
 &= -\$35,000 \text{ (loss)}
 \end{aligned}$$

c. Determine the sales volume as a percentage of capacity to generate a net income of \$61,250.

$$\begin{aligned}
NI &= 61,250 \\
61,250 &= 0.35x - 105,000 \\
61,250 + 105,000 &= 0.35x - 105,000 + 105,000 \\
166,250 &= 0.35x \\
x &= 475,000 \text{ units}
\end{aligned}$$

As a percentage of capacity:

$$\frac{475,000}{500,000} \times 100 = 95\%$$

d. Determine the break-even point in dollars if fixed costs are reduced by \$11,200 while variable costs are changed to 72% of sales.

$$\begin{aligned}
\text{New FC} &= 105,000 - 11,200 = \$93,800 \\
\text{New VC} &= 0.72 \\
\text{New NI} &= x - (0.72x + 93,800) = 0.28x - 93,800
\end{aligned}$$

$$\begin{aligned}
NI &= 0.28x - 93,800 \\
0 &= 0.28x - 93,800 \\
93,800 &= 0.28x \\
x &= 335,000 \text{ units}
\end{aligned}$$

$$TR = \$335,000$$

13. The management of Lambda Corporation has received the following forecast for next year:

Sales Revenue		\$600,000
Fixed Costs	\$275,000	
Variable Costs	270,000	545,000
Net Income		\$55,000

Capacity is a sales volume of \$800,000

Information given:

- Assume $S = \$1$, therefore sales = 600,000 units
- $VC \text{ per unit} = \frac{270,000}{600,000} = 0.45$
- $TR = x$
- $TC = 0.45x + 275,000$

a. Compute the break-even point in dollars and as a percentage of capacity.

$$NI = x - (0.45x + 275,000)$$

$$NI = 0.55x - 275,000$$

$$0 = 0.55x - 275,000$$

$$275,000 = 0.55x$$

$$x = 500,000 \text{ units}$$

$$TR = \$500,000$$

As a percentage of capacity:

$$\frac{500,000}{800,000} \times 100 = \mathbf{62.5\%}$$

b. Determine the net income at 50% of capacity.

$$50\% \text{ of capacity} = 0.50(800,000) = 400,000$$

$$NI = 0.55x - 275,000$$

$$= 0.55(400,000) - 275,000$$

$$= 220,000 - 275,000$$

$$= -\$55,000$$

c. Determine the sales volume in dollars to generate a net income of \$82,500.

$$NI = 82,500$$

$$82,500 = 0.55x - 275,000$$

$$82,500 + 275,000 = 0.55x - 275,000 + 275,000$$

$$357,500 = 0.55x$$

$$x = 650,000 \text{ units}$$

$$TR = \$650,000$$

d. Determine the break-even volume in dollars if fixed costs are increased by \$40,000 while variable costs are held to 40% of sales.

$$\text{New FC} = 275,000 + 40,000 = \$315,000$$

$$\text{New VC} = 0.40$$

$$\text{New NI} = x - (0.40x + 315,000) = 0.60x - 315,000$$

$$0 = 0.60x - 315,000$$

$$315,000 = 0.60x$$

$$x = 525,000 \text{ units}$$

$$TR = \$525,000$$

14. Alpha Corp. expects to operate at 80% capacity next year. Its forecast operating budget is:

Sales Revenue		\$1,200,000
Fixed Costs	\$300,000	
Variable Costs	800,000	
Total Costs		1,100,000
Net Income		\$100,000

Information given:

- Let S (Sale price) = \$1 and X represents the number of \$1 units sold. → Then TR = \$1X
- TR = \$1,200,000 therefore sales=1,200,000 units
- $VC \text{ per unit} = \frac{800,000}{1,200,000} = 0.6666667$
- $TC = VCx + FC = 0.6666667x + 300,000$

a. What is Alpha's break-even revenue?

$$\begin{aligned}
 NI &= x - (0.6666667x + 300,000) \\
 &= 0.333333x - 300,000 \\
 0 &= 0.333333x - 300,000 \\
 300,000 &= 0.333333x \\
 x &= 900,000
 \end{aligned}$$

$$TR = \$900,000$$

b. What would be Alpha's net income if it operates at full capacity?

$$\begin{aligned}
 &80\% \text{ capacity has revenue of } \$1,200,000 \\
 &\frac{1,200,000}{0.80} = \frac{\text{full capacity}}{1.00} \\
 &\frac{1,200,000}{0.80} = \$1,500,000, \text{ which is } 1,500,000 \text{ units}
 \end{aligned}$$

$$\begin{aligned}
 NI &= 0.333333x - 300,000 \\
 &= 0.333333(1,500,000) - 300,000 \\
 &= 500,000 - 300,000 \\
 &= \$200,000
 \end{aligned}$$